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ON THE ANALOGY BETWEEN THE WAVE MOTIONS OF CHEMICALLY ACTIVE AND TWO-PHASE MEDIA*

A.L. Ni

Nonlinear asymptotic equations that describe the wave motions of two-phase media consisting of a gas and solid heavy particles suspended in it are derived. Within the framework of the theory developed, it is shown that there exists an exact mathematical analogy between the motions of two-phase and chemically active media. By means of this analogy, it is possible to pass from well-known results for chemically active media to two-phase systems without making any changes.

Laws governing the behavior of pressure waves in chemically active media have now been rather well-studied. It has been established /1/ that the propagation of acoustic pulses is accompanied here by dispersion of the rate of transmission of the signals, which lies between the "frozen" and equilibrium speeds of sound and coincide with them in the limiting cases of high- and low-frequency waves, respectively. In the linear approximation, the influence of dispersion on lengthy periods of time is equivalent to the effect of bulk viscosity. That is, the fact that the chemical composition is not in equilibrium leads to anomalous expansion of the shock wave front. Nonlinear analysis /2/ gives more meaningful information about the evolution of weak shock waves and rarefaction waves. The results of /2/ were generalized in /3,4/ for matter with an arbitrary number of chemical reactions. The analogy between the wave motions of chemically active and two-phase media within the framework of linear equations has also been discussed in /5 – 7/. Nonlinear analysis of slightly perturbed supersonic two-phase currents has also been conducted /5/. The approach used here was proposed in /8/ in connection with the study of nonequilibrium currents. However, other results /5/ lack a degree of visualization, since all the studies were conducted using semi-characteristic variables, while the kinetic equations are linear; the role of nonlinearity is seen in the transition to the physical plane, which may be undertaken by means of finite formulas only in small regions of the current.

1. Equations of motion. We neglect the influence of the particles on each other and their natural volume. We also assume that the density of the material of the particles ρ_s° is much greater than the density of the gas ρ, and that intra-phase interaction reduces to the reciprocal friction of the gas and particles exclusively. Heat exchange between the phases will not be considered, for the sake of simplicity. As will clearly follow from the succeeding arguments, the inclusion of heat exchange into a model within the framework of small perturbations is equivalent to the introduction of one more relaxational parameter and may be taken into account as in /3,4/. The equations of motion of the mixture may be taken in the form /9/

$$\frac{\partial\rho_s}{\partial t}+\frac{\partial\rho_s u_s}{\partial r}+\frac{\nu-1}{r}\rho_s u_s=0,\quad \frac{\partial\rho}{\partial t}+\frac{\partial\rho u}{\partial r}+\frac{\nu-1}{r}\rho u=0 \tag{1.1}$$
$$\rho_s\frac{\partial u_s}{\partial t}+\rho_s u_s\frac{\partial u_s}{\partial r}=-f,\quad \rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial r}=f-\frac{\partial p}{\partial r}$$
$$T\left(\frac{\partial s}{\partial t}+u\frac{\partial s}{\partial r}\right)=-\frac{f}{\rho}(u-u_s),$$
$$\frac{\partial n}{\partial t}+\frac{\partial n u_s}{\partial r}+\frac{\nu-1}{r}n\,{}_s=0$$

(the last equation of the system is a simple corollary of the equation for the conservation of the mass of the particles). Here r is the distance from the center, axis or plane of symmetry, and u and ρ are the speed and density of the gas; the subscript "s" indicates the corresponding variables referred to the particle gas; p is the pressure; T is the temperature; s is the specific entropy of the gas; and n is the number of particles per unit of volume. The parameter $\nu=1,2,3$ for currents with a plane, axis, or center of symmetry, respectively.

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The functional relations $p = p(\rho, s)$, $T = T(\rho, s)$ are the missing equations of the state of the gas.

To close the system (1.1), it is necessary to determine the bulk force of the intra-phase friction. For the sake of simplicity, we will assume that the particles are spheres with radius a. Then the interaction force f between a solid particle and the nonstationary gas flow may be represented in the form of a sum of the Stokes force, Basset force, and inertial force, which takes into account the influence of added masses /10/. Simple estimates show that the last two forces must be taken into account in the equations of motion together with the Stokes force at times on the order of $a^2/p\eta$ (η is the dynamic viscosity coefficient of the gas). But at such times the variation of the velocity of the particles, for example, due to interaction with the gas is on the order of $u\rho/\rho_s^\circ$ (u is the velocity of the particle relative to the gas). This means that the basic contribution to the intra-phase interaction in this case is provided by the Stokes force; by taking into account the Basset force and added masses, we are led to make small corrections on the order of ρ/ρ_s. Note that it is precisely the Basset force which has preferred /11/ for analyzing the wave motions of gas mixtures; as was shown above, such an assumption is invalid for heavy particles. Finally, we set

$$f = n\alpha(u_s - u), \quad \alpha = 6\pi\eta a$$

2. Speed of sound. Before passing to the derivation of the basic nonlinear equations that describe the wave motion of a medium, we will study certain properties of the linearized system (1.1). Everywhere below the zero subscript will denote parameters of the medium in the initial quiescent state, and the prime will indicate deviations of the parameters of the mixture from the unperturbed state. Linearization of (1.1) yields ($\nu = 1$)

$$\frac{\partial \rho_s'}{\partial t} + \rho_{s0}\frac{\partial u_s'}{\partial r} = 0, \quad \frac{\partial \rho'}{\partial t} + \rho_0\frac{\partial u'}{\partial r} = 0$$

$$\rho_{s0}\frac{\partial u_s'}{\partial t} = -n_0\alpha(u_s' - u'), \quad \rho_0\frac{\partial u'}{\partial t} = -a_0^2\frac{\partial \rho'}{\partial r} - \left(\frac{\partial p}{\partial s_0}\right)_\rho \frac{\partial s'}{\partial r} +$$

$$n_0\alpha(u_s' - u'), \quad \frac{\partial s'}{\partial t} = 0, \quad a_0^2 = \left(\frac{\partial p}{\partial \rho_0}\right)_s$$

Substitution of the resulting solution in the form

$$\mathbf{A} = \mathbf{A}_0 e^{i(\omega t - kr)}, \quad \mathbf{A} = (u_s', \rho_s', u', \rho', s')$$

where $\mathbf{A}_0$ is a constant column vector and subsequently setting the determinant of the resulting system of linear algebraic equations equal to zero yields the characteristic equation

$$\omega^2\left[\omega^3 + \frac{n_0\alpha}{i}\omega^2\left(\frac{1}{\rho_{s0}} + \frac{1}{\rho_0}\right) - k^2a^2\left(\omega + \frac{n_0\alpha}{i\rho_{s0}}\right)\right] = 0$$

The multiple root $\omega = 0$ corresponds to the paths of motion of the gas and particles. Setting the expression within the brackets equal to zero, we obtain in the limiting cases

$$\omega = \pm ka_0 \quad \text{if} \quad \omega \gg \max(n_0\alpha/\rho_0,\ n_0\alpha/\rho_{s0}) \tag{2.1}$$

$$\omega = \pm ka_0\sqrt{\rho_0/(\rho_0 + \rho_{s0})} \quad \text{if} \quad \omega \ll \min(n_0\alpha/\rho_0,\ n_0\alpha/\rho_{s0}) \tag{2.2}$$

Thus, we conclude that in a two-phase medium transmission of signals is accompanied by dispersion; the rate of propagation of the acoustic pulses coincides, in the limiting cases, either with the frozen rate of sound a_f or with the equilibrium speed a_e, further

$$a_f = \sqrt{(\partial p/\partial \rho)_s}, \quad a_e = \sqrt{\rho/(\rho_s + \rho)}\, a_f \leqslant a_f$$

Below we will use precisely such a terminology to denote the corresponding speeds of sound. In a rapid high-frequency wave the solid particles are unable to follow the motion of the gas; in a slow low-frequency wave the velocity of the particles is equal to its equilibrium value, or the velocity of the gas. Here the profound analogy between wave motions in two-phase and relaxational mixtures is obvious, further the velocity of the particles in two-phase media is equivalent to the completion of a chemical reaction /2/. A detailed survey of studies on the acoustics of heterogeneous media may be found in /7/.

3. Asymptotic expansions. We now study different limiting modes of propagation of perturbations in two-phase media using the expansion of the required functions in a series in several small parameters.

Everywhere below we will assume that the values of the characteristics of the current differ little from the corresponding values for the quiescent state. We introduce a coordinate system moving with velocity c_0 whose value will be selected as a function of the particular situation.

We will suppose that the current in the medium may be represented as a short wave, i.e., the width of the disturbed region is small by comparison with the distances at which the wave propagates. Accordingly, we introduce dimensionless coordinates by the formulas

$$t = \frac{L}{\Delta c_0} t', \quad r = c_0 t + L r' \tag{3.1}$$

where L is the characteristic dimension of the flow in a movable coordinate system and Δ is a small parameter.

Let the small parameter ε determines the amplitude of the disturbances propagating in the mixture. Then, passing to dimensionless dependent variables yields

$$\begin{aligned} &u_s = \varepsilon c_0 u_s', \ \rho_s = \rho_{s0}(1 + \varepsilon\rho_s'), \ u = \varepsilon c_0 u' \\ &\rho = \rho_0(1 + \varepsilon\rho'), \ p = p_0(1 + \varepsilon p'), \ s = s_0(1 + \varepsilon s') \\ &a_f = a_{f0}(1 + \varepsilon a_f'), \ a_e = a_{e0}(1 + \varepsilon a_e') \end{aligned} \tag{3.2}$$

Further, we will use equations that are linear combinations of the second, fourth and fifth equations of the system (1.1):

$$\begin{aligned} &\frac{\partial p}{\partial t} + (u + a_f)\frac{\partial p}{\partial r} + \rho a_f\left[\frac{\partial u}{\partial t} + (u + a_f)\frac{\partial u}{\partial r} + (\nu - 1)\frac{a_f u}{r}\right] = \\ &\quad a_f f + a_f^2\left(\frac{\partial \rho}{\partial s}\right)_p \frac{f(u - u_s)}{\rho T} = L_f \\ &\frac{\partial p}{\partial t} + (u + a_e)\frac{\partial p}{\partial r} + \rho a_e\left[\frac{\partial u}{\partial t} + (u + a_e)\frac{\partial u}{\partial r}\right] + \\ &\quad (\nu - 1)\rho\frac{a_f^2 u}{r} = a_e f - (a_f^2 - a_e^2)\rho\frac{\partial u}{\partial r} + \\ &\quad a_f^2\left(\frac{\partial \rho}{\partial s}\right)_p \frac{f(u - u_s)}{\rho T} = L_e \end{aligned} \tag{3.3}$$

4. Quasi-frozen mode. Everywhere below the prime will be omitted from the dimensionless variables. We will assume that the equilibrium and frozen speeds of sound differ by a finite magnitude or, what is the same thing, $\rho_{s0} \sim \rho_0$. Let us consider the frozen short wave that propagates, in a linear approximation, through the quiescent medium with velocity a_{f0}. Accordingly, in (3.1) and (3.2) we set $c_0 = a_{f0}$ and substitute them in the equations of motion (1.1). Then, ignoring higher order of smallness and integrating the resulting linear equations, we arrive at the formulas

$$\rho = \frac{p_0}{\rho_0 a_{f0}^2} p, \quad \rho = u, \quad \rho_s = u_s \tag{4.1}$$

The first equation in (4.1) indicates that gas compression is reversible. The second equation is analogous to the Riemann relation in a simple wave. Note that formulas (4.1) do not contradict the energy equation, since in this approximation $s = 0$. With the last equation, we find that

$$a_f = \left(\frac{\partial a_f}{\partial \rho_0}\right)_s \frac{\rho_0}{a_{f0}} \rho = (m_0 - 1)u, \quad m_0 = \frac{1}{2\rho_0^3 a_{f0}^2}\left(\frac{\partial^2 p}{\partial V_0^2}\right)_s, \quad V_0 = \frac{1}{\rho_0}$$

The equation for the velocity of the particles in dimensionless variables is described thus

$$\frac{\partial u_s}{\partial r} = N_r(u_s - u), \quad N_r = \frac{n_0 \alpha L}{a_{f0}\rho_{s0}}$$

In the quasi-frozen mode, we find that $N_r \ll 1$ in accordance with inequalities (2.2). Then from the last equation it follows that $u_s = 0$. Retaining the principal terms in the first equation of (3.3) and using (4.1) as well as an expression for the increment in the speed of sound a_f, we obtain the nonlinear equation

$$2\varepsilon m_0 u\frac{\partial u}{\partial r} + \Delta\left[2\frac{\partial u}{\partial t} + (\nu - 1)\frac{u}{t}\right] = \frac{n_0 \alpha L}{a_{f0}\rho_0}(u_s - u) = -\frac{\rho_{s0}}{\rho_0} N_r u \tag{4.2}$$

which is entirely analogous, in terms of structure, to equation (3.5) in /2/, which describes the quasi-frozen wave in a chemically active mixture. In the plane case, for example, it yields a solution that attenuates along both characteristics according to an exponential law.

5. Quasi-equilibrium mode. To study a quasi-equilibrium short wave, we set $c_0 = a_{e0}$ in (3.1) and (3.2) in accordance with the conclusions of the linear theory. We may verify that (4.1) are true also for the quasi-equilibrium situation.

The equation for the velocity of the particles may be reduced to the form

$$\frac{\partial u_s}{\partial r} = N_e (u_s - u), \quad N_e = \frac{n_0 \alpha L}{a_{e0} \rho_{s0}} \tag{5.1}$$

further that $N_e \gg 1$, as follows from (2.3). Substituting (5.1) in the equation for the increment in the entropy yields $s \sim \varepsilon / N_e$. Recalling the definition of the equilibrium speed of sound, we find after some transformation that

$$a_e = a_f + \frac{1}{2} \frac{\rho_{s0}}{\rho_{s0} + \rho_0} (\rho - \rho_s) \tag{5.2}$$

From equation (5.1) and the condition $N_e \gg 1$, we have $u_s = u + O(1/N_e)$. Hence, from (4.1) we find that $a_e = a_f$ to within high-order infinitesimals.

To derive the nonlinear quasi-equilibrium wave equation, we will use the second equation of (3.3). Substituting (3.1), (3.2), (4.1), and (5.2) in it and discarding high-order infinitesimals, we obtain

$$2 \varepsilon m_0 u \frac{\partial u}{\partial r} + \Delta \left[2 \frac{\partial u}{\partial t} + (\nu - 1) \frac{u}{t} \right] = \frac{\rho_{s0}}{\rho_0} \frac{a_{e0}^2}{a_{f0}^2} \frac{1}{N_e} \frac{\partial^2 u}{\partial r^2}$$

The last equation is entirely identical to (4.2) of /2/, where it was obtained for the purpose of describing quasi-equilibrium waves in chemically active media. On the other hand, equation (5.3) was derived and studied /12–14/ for the analysis of the flow of a neutral gas that exhibits viscosity and heat conductivity.

The equation studied in /12–14/ turns into (5.3) if a correspondence is established between the coefficients in accordance with the following rule

$$\frac{\rho_{s0}}{\rho_0} \frac{a_{e0}^2}{a_{f0}^2} \frac{1}{N_e} \to \frac{1}{\mathrm{Re}} \left(1 + \frac{\varkappa - 1}{\mathrm{Pr}} \right)$$

$$\mathrm{Re} = \left(\frac{4}{3} \frac{\eta}{\rho_0 a_0 L} + \frac{\zeta}{\rho_0 a_0 L} \right)^{-1}, \quad \mathrm{Pe} = \frac{\rho_0 a_0 c_p L}{k}, \quad \mathrm{Pr} = \frac{\mathrm{Pe}}{\mathrm{Re}}, \quad a_0^2 = \left(\frac{\partial p}{\partial \rho} \right)_s$$

Here ζ is the second viscosity coefficient; k, heat conductivity coefficient; c_p is specific heat capacity at constant pressure and $\varkappa$ is Poisson adiabatic index. Hence follows an exact mathematical analogy between the two processes. According to this analogy, the influence of the relative motion of the gas and particles on the quasi-equilibrium propagation of nonlinear acoustic pulses leads to an effect equivalent to the action of longitudinal viscosity and heat conductivity. Simples estimates show that the effective viscosity of a two-phase medium determined by the quantity N_e may markedly exceed the natural viscosity of the gas, as determined by the coefficient Re.

6. Media with nearly equal speeds of sound. Below we will derive an equation that is uniformly suitable for the description of the propagation of acoustic pulses in a two-phase medium. It incorporates both cases described in Sects.4 and 5 as limiting cases and may be used to describe the continuous transition from one mode to the next. To properly derive a uniformly suitable equation, we will assume that the equilibrium and frozen speeds of sound are nearly equal or, what is the same thing, that $\rho_{s0}/\rho_0 \ll 1$. The difference between the two speeds of sound will be characterized by the small parameter ε_a^2.

Let us introduce a coordinate system moving with speed c_0 and determined by the formula

$$c_0 - a_{f0} = \varepsilon_a^2 \sigma_{f0} c_0, \quad c_0 - a_{e0} = \varepsilon_a^2 \sigma_{e0} c_0 \tag{6.1}$$

where σ_{f0} and σ_{e0} have the order of unity.

Linearization of equations (1.1) again leads to (4.1).

We have the estimate $s \sim \varepsilon \varepsilon_a^2$ from the equation for the increment in entropy.

The equation for the velocity of the particles in dimensionless variables is re-written thus:

$$\partial u_s / \partial r = N_r (u_s - u) \tag{6.2}$$

Note that in this approximation N_r and N_e coincide to within high-order of smallness. To derive the missing relations, we return again to equations (3.3).

The first of the relations from (3.3) yields (following some simplifications)

$$2 (\varepsilon m_0 u - \varepsilon_a^2 \sigma_{f0}) \frac{\partial u}{\partial r} + \Delta \left[2 \frac{\partial u}{\partial t} + (\nu - 1) \frac{u}{t} \right] = \frac{\rho_{s0}}{\rho_0} N_r (u_s - u) \tag{6.3}$$

Analogously, from the second relation we find that

$$2(\varepsilon m_0 u - \varepsilon_a{}^2\sigma_{e0})\frac{\partial u}{\partial r} + \Delta\left[2\frac{\partial u}{\partial t} + (\nu - 1)\frac{u}{t}\right] = \frac{\rho_{s0}}{\rho_0} N_r (u_s - u) - \frac{\rho_{s0}}{\rho_0}\frac{\partial u}{\partial r} \tag{6.4}$$

Equations (6.3) and (6.5) are equivalent. In fact, it follows from (6.1) that

$$\varepsilon_a{}^2(\sigma_{f0} - \sigma_{e0})c_0 = a_{e0} - a_{f0}$$

Hence, recalling the definition of a_e and a_f and using the inequality $\rho_{s0} \ll \rho_0$, we find that

$$\varepsilon_a{}^2\sigma_{f0} = \varepsilon_a{}^2\sigma_{e0} - \frac{1}{2}\rho_{s0}/\rho_0$$

Substitution of this expression into (6.3) reduces it to the form (6.4).

Eliminating from (6.2) and (6.3) the velocity of the particles u_s, we obtain a single second-order equation in the unknown function u:

$$\frac{\partial}{\partial r}\left\{2(\varepsilon m_0 u - \varepsilon_a{}^2\sigma_{f0})\frac{\partial u}{\partial r} + \Delta\left[2\frac{\partial u}{\partial t} + (\nu - 1)\frac{u}{t}\right]\right\} -$$
$$N_r\left\{2(\varepsilon m_0 u - \varepsilon_a{}^2\sigma_{e0})\frac{\partial u}{\partial r} + \Delta\left[2\frac{\partial u}{\partial t} + (\nu - 1)\frac{u}{t}\right]\right\} = 0$$

From (6.5), we obtain the limiting modes (4.2) and (5.3). Suppose that $\sigma_{f0} = 0$ and $\varepsilon_a{}^2N_r \sim \varepsilon$, $N_r \ll 1$. Retaining in (6.5) the principal terms and integrating it once, we arrive at (4.2). If $\sigma_{e0} = 0$ and $\varepsilon_a{}^2/N_e \sim \varepsilon$, $N_e \gg 1$, then (6.5) turns into (5.3). Equations (6.2) and (6.3), along with (6.5), are entirely analogous to previous equations /2/ obtained for the description of short waves in chemically active mixtures. Therefore, all the conclusions regarding waves in chemically active mixtures may be carried over to two-phase systems without any changes.

Let us briefly list the basic conclusions. In two-phase media, two types of shock waves are possible: waves with complete dispersion propagate at speeds of $a_{e0} < D < a_{f0}$, and compression in such waves is realized continuously; waves with frequency dispersion have speeds $D > a_{f0}$ and the disturbance zone in such waves is bounded by a front at which the parameters of the particles remain unchanged, but at which the quantities that describe the gas are discontinuous. The solution for two-phase mixtures may be described by analogy with the problem of a centered rarefaction wave in a gas briefly as follows. At the initial stages, where $N_r \ll 1$ the particles are not pulled along by the gas and the solution is essentially that of a rarefaction wave in the pure gas. When $N_e \gg 1$ in the basic current region, except for the boundary layers, the velocity of the particles is equal to the velocity of the gas and the acceleration of the mixture is realized in a centered equilibrium rarefaction wave. In the interval between the leading equilibrium and frozen characteristics the parameters of the mixture tend exponentially with time to their values in the quiescent state.

The existence of partially and completely dispersed shock waves in gas and liquid systems and its experimental verification have been demonstrated /6/.

Above we considered solely the influence of dynamic slippage of solid particles relative to a gas on the wave processes in two-phase systems, since in this approximation the temperature disequilibrium of the medium is entirely equivalent to the effect of a chemical reaction /2/ (under the condition that the Newton–Rikhman law is valid for the description of intra-phase heat exchange). In light of the foregoing, we may conclude that the combined effect of dynamic slippage and intra-phase heat exchange is equivalent, within the framework of this approach, to the introduction of two chemical reactions, /3,4/. Depending upon the relation between the characteristic times of the temperature and velocity relaxation it is possible for the wave packets to separate into time layers and for the shock waves to have a band structure /15/. (*) See next page.

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*) In the original, on page No.795 (in translation p.No.635), the numbers used in the text are (6.5) and (5.3), but no formulas are marked with these figures - Ed.